CHI-SQUARED TEST

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1. INTRODUCTION

Data analysis that incorporates hypothesis testing includes: association analysis (Chi-Square, Pearson´s Correlation), comparing means (One Sample *t* Test, Paired Sample *t* Test…) and predictive models (Multiple Regression, Logistic Regression…). In this section we are going to focus on the association analysis of **Chi-squared test**: how to calculate and interpret it.

2. CHI-SQUARED (χ^2) TEST

There are two types of chi-square tests:

- *Chi-square goodness of fit test*, which determines if sample matches a population. (https://www.scribbr.com/statistics/chi-square-goodness-of-fit/)
- *Chi-square test of independence*, that determines whether two categorical variables appear to be associated or related using a contingency table (cross-tabulation). (https://www.scribbr.com/statistics/chi-square-test-of-independence/)

The chi-square test of independence is the most commonly used, and it can only compare categorical variables:

- Nominal: variables with no inherent order or rank. (E.g. gender, race, university…).
- Ordinal: variable with an order or rank. (E.g. blood group, performance…).
- Binary: variables with only two possible options (E.g. pass/fail, yes/no…).

To perform a chi-squared test, first you must determine the hypothesis:

- The null hypothesis (H_0): [*Variable 1*] is not associated with [*Variable 2*].
- The alternative hypothesis (H_1): [*Variable 1*] is associated with [*Variable 2*].

Chi-squared test will give you a p-value, that will tell you if your test results are significant or not.

3. HOW TO CALCULATE CHI-SQUARE

a. By hand

The formula used in chi-squared test is:

$$\chi^2 = \sum \frac{(Obs. - Exp.)^2}{Exp.}$$

- *Obs.*: is the observed value
- *Exp.*: is the expected value

Example: 478 students conducted a survey to find out their zodiac sign. The results were: Aries (48), Taurus (25), Gemini (64), Cancer (19), Leo (55), Virgo (21), Libra (30), Scorpio (72), Sagittarius (18), Capricorn (67), Aquarius (39) and Pisces (20). Test the hypothesis that zodiac signs are equally distributed across students.

1) Make a table with the columns for "Categories", "Observed", "Expected", "(Residual=(Obs-Exp)", "$(Obs-Exp)^2$" and "$Component=(Obs-Exp)^2/Exp$".

2) Fill in the categories and do the calculations.
Categories: zodiac signs.
Observed: numbers of students of each zodiac sign.
Expected: if we have 478 people enrolled in the survey, we expect 478 divided by 12 (zodiac signs).

3) Add up all the values on the "Component" column.

Categories	Observed	Expected	Residual=(Obs-Exp)	$(Obs-Exp)^2$	$Component=(Obs-Exp)^2/Exp$
Aries	48	39,833	8,167	66,700	1,674
Taurus	25	39,833	-14,833	220,018	5,523
Gemini	64	39,833	24,167	584,044	14,662
Cancer	19	39,833	-20,833	434,014	10,896
Leo	55	39,833	15,167	230,028	5,775
Virgo	21	39,833	-18,833	354,682	8,904
Libra	30	39,833	-9,833	96,694	2,427
Scorpio	72	39,833	32,167	1034,694	25,976
Sagittarius	18	39,833	-21,833	476,680	11,967
Capricorn	67	39,833	27,167	738,028	18,528
Aquiarius	39	39,833	0,833	0,694	0,017
Pisces	20	39,833	-19,833	393,348	9,875
					116,225

116,225 is our chi-square statistic.

As the calculations can be very tedious and lengthy, instead you can use SPSS computer program.

The p-value is calculated with the chi-squared statistic (116,225) and the degrees of freedom (nº of categories – 1= 12-1=11). Using an online chi-squared p-value calculator we can find out or value.
(https://www.socscistatistics.com/pvalues/chidistribution.aspx)

Chi-square score: 116,22
DF: 11

Significance Level:

- ○ 0.01
- ◉ 0.05
- ○ 0.10

The P-Value is < .00001. The result is significant at p < .05.

As the value is <0,00001, we can state that there is strong evidence against the null hypothesis, and we will have to reject it.

Example 2, using a scientific publication: A study reported a randomized trial comparing the incidence of bradycardia after phenylephrine versus norepinephrine to prevent and treat spinal-induced hypotension in women undergoing cesarean delivery with spinal anesthesia. The authors used a chi-square ($\chi 2$) test to compare the groups and observed a lower incidence of bradycardia in the norepinephrine group. (https://journals.lww.com/anesthesia-analgesia/fulltext/2019/11000/chi_square_tests_in_medical_research.3.aspx)

[redacted]

			Treatment Group		
			Phenylephrine	Norepinephrine	Total
Bradycardia	Yes	Observed Number	21	6	27
		Expected Number	*13.5*	*13.5*	
	No	Observed Number	35	50	85
		Expected Number	*42.5*	*42.5*	
	Total		56	56	112

[redacted]

Figure.

Contingency table with data from Sharkey et al[1] showing the observed and expected counts (number of patients) with and without bradycardia per treatment group. Assuming that the probability of developing bradycardia is independent of the group allocation (null hypothesis), 13.5 patients with bradycardia would be expected in each group (given a total of 27 patients who developed bradycardia and equal sample size in both groups). Pearson χ^2 test compares observed to expected frequencies.

Source

Chi-square Tests in Medical Research

Anesthesia & Analgesia129(5):1193, November 2019.

Categories	Observed	Expected	Residual=(Obs-Exp)	$(Obs-Exp)^2$	$Component=(Obs-Exp)^2/Exp$
Bradychardia with phenylephrine	21	13,500	7,500	56,250	4,167
Bradychardia with norepinephrine	6	13,500	-7,500	56,250	4,167
No bradychardia with phenylephrine	35	42,500	-7,500	56,250	1,324
No bradychardia with norepirephrine	50	42,500	7,500	56,250	1,324
					10,980

10,980 is our chi-square statistic.

Chi-square score: 10,980

DF: 3

Significance Level:

- ○ 0.01
- ◉ 0.05
- ○ 0.10

The P-Value is < .00001. The result is significant at p < .05.

As the value is <0,00001, we can state that there is strong evidence against the null hypothesis, and we will have to reject it.

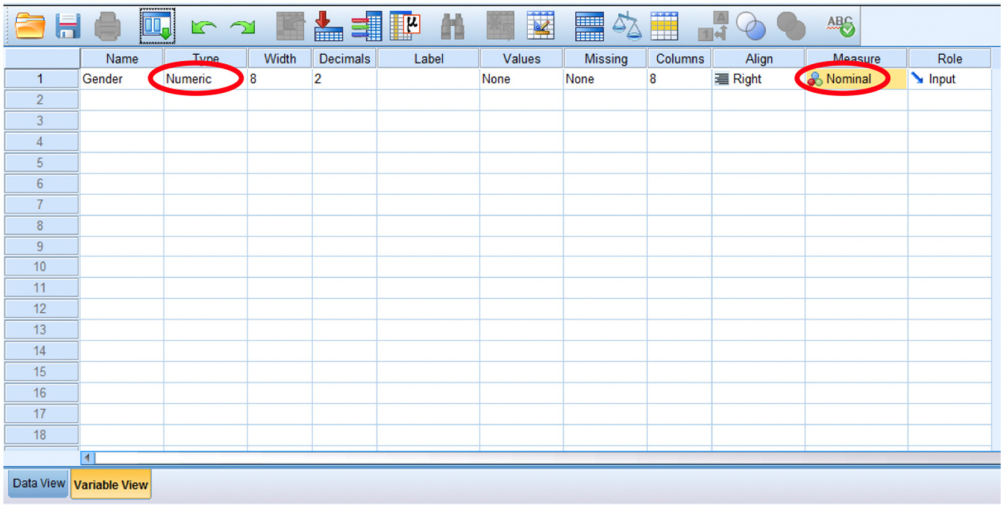
b. By SPSS

Download SPSS and have a 30-day trial free trial here: https://www.ibm.com/es-es/spss

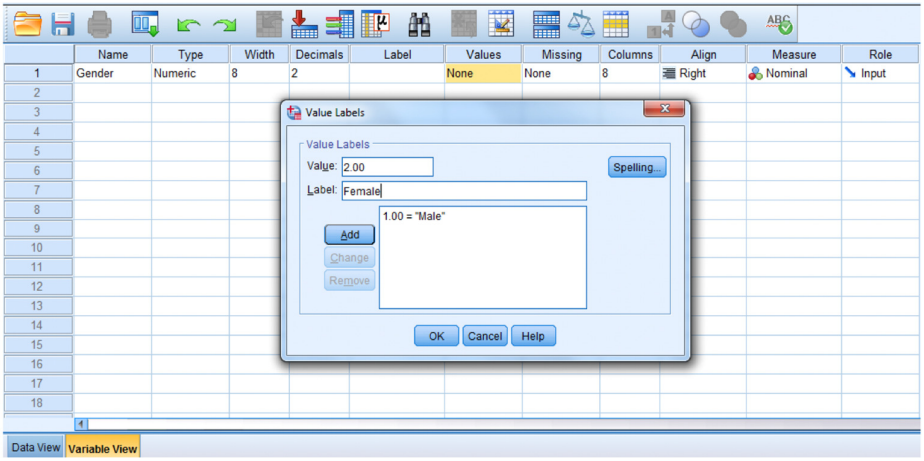
We have conducted a survey asking people on the University whether they live in student halls or with their families during the term. These are the results:

	Where do you live in term time?		Total
	Halls	Family home	
Female	14	10	24
Male	19	7	26
Total	33	17	**50**

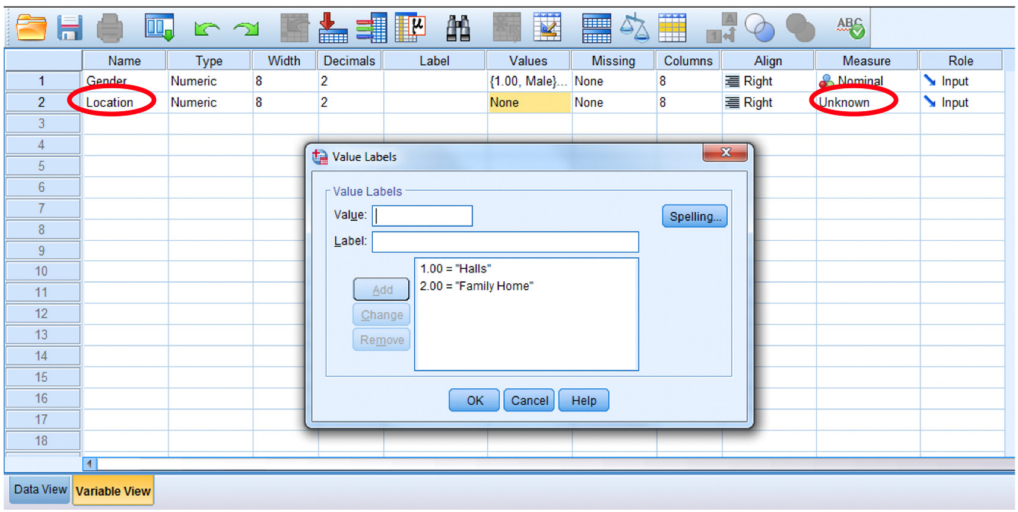
1) Click variable view, where you can define columns of data that you are about to enter. Start with the first variable (Gender).



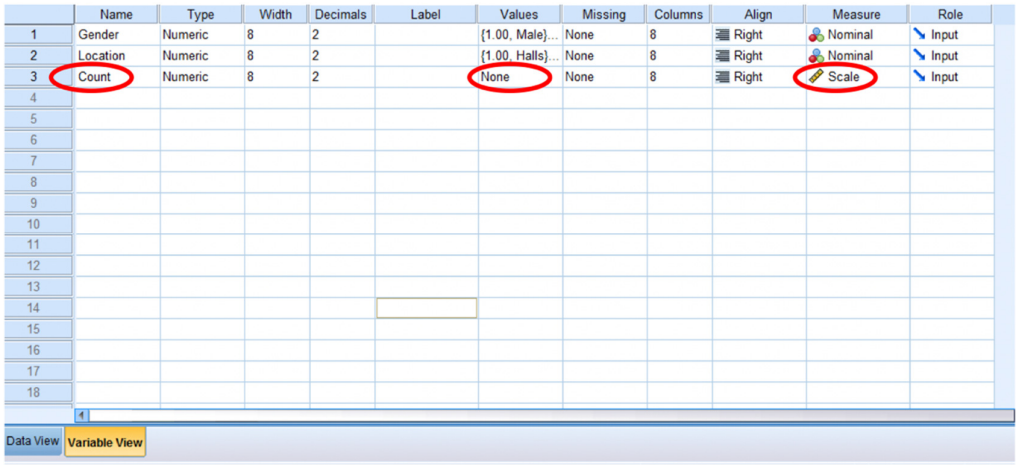
2) Define values and their labels, where 1 is Male and 2 is Female.



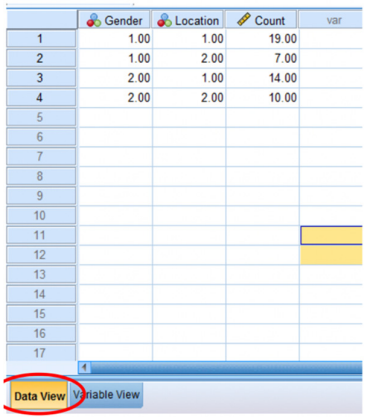
3) Add another row in the SPSS file: “location” where 1 represents hall and 2 represents family home.



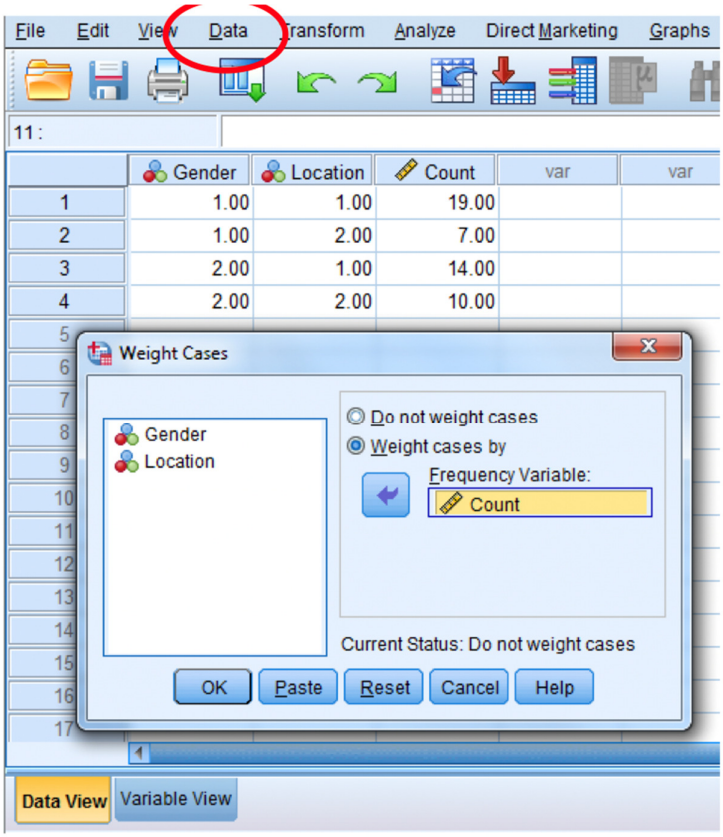
4) Add a row in the SPSS file: “count” measuring the number of people surveyed.



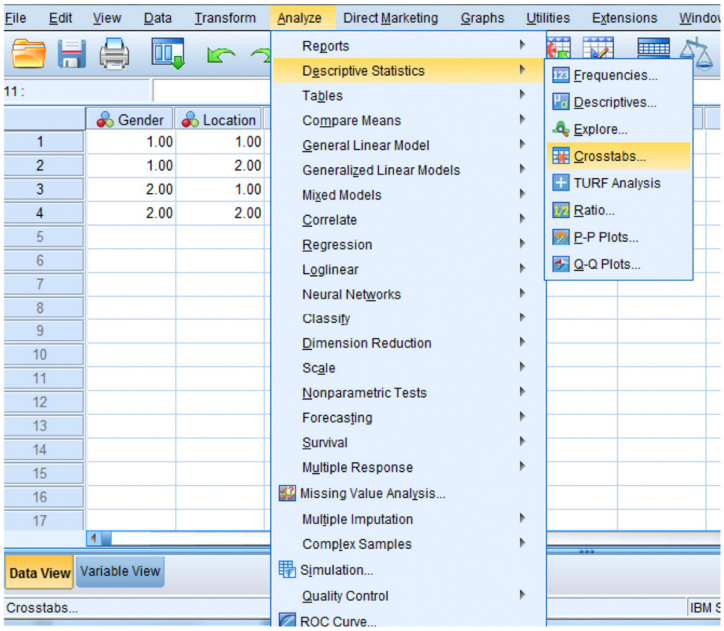
5) Within `Data View´ add your total values for each of your 4 groups in the data tab (take counts from your data gathering).



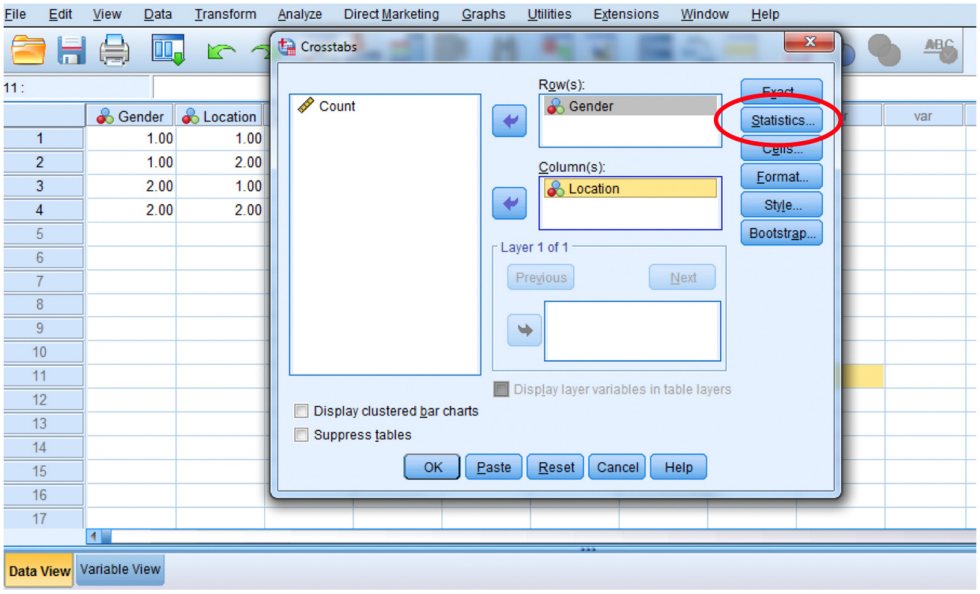
6) Weight your data correctly: Data- Weight cases.

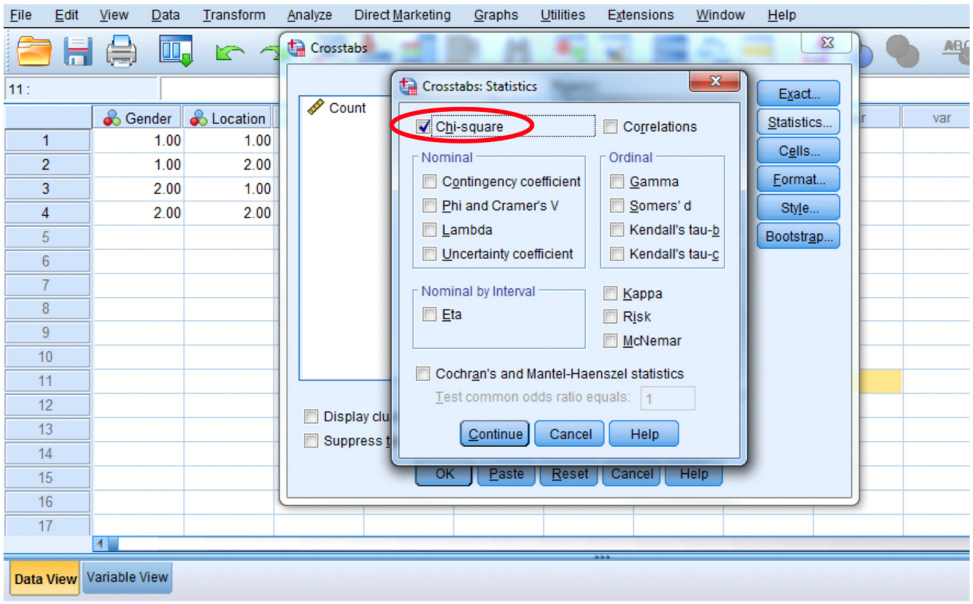


7) Go to: Analyze- Descriptive statistics- Crosstabs.

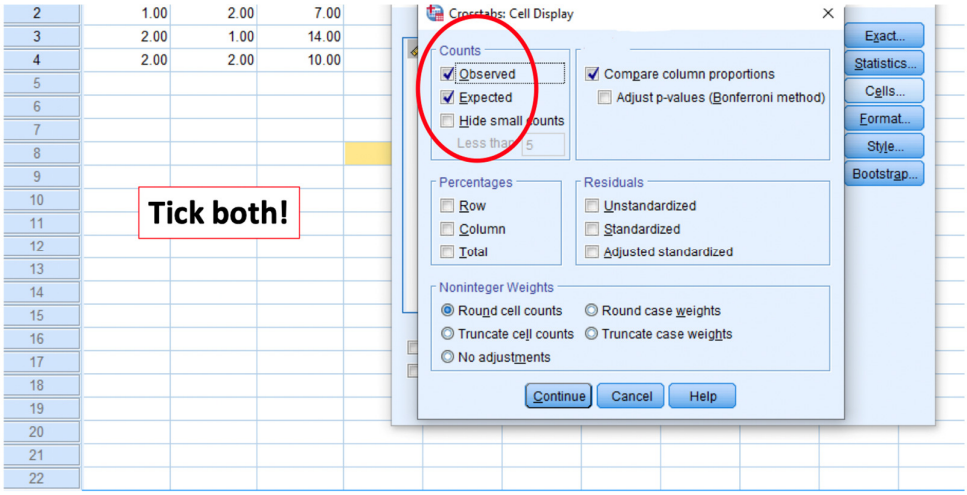


8) Click `Statistics´, then Chi-square and continue.





9) Click on `Cells´and tick the boxes: observed and expected.



10) See the results.

Gender * Location Crosstabulation

			Location		
			Halls	Family	Total
Gender	Male	Count	19	7	26
		Expected Count	17.2	8.8	26.0
	Female	Count	14	10	24
		Expected Count	15.8	8.2	24.0
Total		Count	33	17	50
		Expected Count	33.0	17.0	50.0

Chi-Square Tests

	Value	df	Asymptotic Significance (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	1.209[a]	1	.272		
Continuity Correction[b]	.641	1	.423		
Likelihood Ratio	1.213	1	.271		
Fisher's Exact Test				.373	.212
Linear-by-Linear Association	1.185	1	.276		
N of Valid Cases	50				

a. 0 cells (,0%) have expected count less than 5. The minimum expected count is 8,16.

b. Computed only for a 2x2 table

11) Interpretation of results.

Our **p-value is 0,272**, we reject the null hypothesis that there's no difference between the variables and conclude that a significant difference does exist.

Here you have an explanatory video with another example:

https://www.youtube.com/watch?v=A_oV3MB53Y0&t=2s